Small-Scale Turbulence and Mixing: Energy Fluxes in Stratified Lakes

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Introduction

Density Stratification and Mixing – the Basin Scale

Nearly all lakes, reservoirs, and ponds that are deeper than a few meters, experience cycles of density stratification and destratification. Most important for this variation is the temperature-dependence of water density. During spring/summer – or the wet season in the tropics – the water is heated from above and a surface layer (SL: typically a few m thick) with warmer and hence lighter water develops on top of the cooler and heavier water below (Figure 1). In addition, although more important in saline lakes than freshwater ones, biological and hydrological processes may strengthen the density stratification by generating a vertical gradient in the concentration of dissolved substances (salinity). The resulting stratification is usually depicted by a strong density gradient (also called pycnocline), separating the SL from the deeper reaches of the water column (indicated as metalimnion and hypolimnion in Figure 1). Mixing of heavier water from greater depth with lighter water from the SL implies that water parcels of different densities are exchanged in the vertical direction (Figure 2). It is evident that mechanical energy is needed to move these water parcels against the prevailing density gradient, which forces lighter water up and heavier water down. The amount of energy needed to overcome vertical density stratifications is therefore determined by the potential energy ΔE_{pot} (Figure 1) stored in the stratification. ΔE_{pot} is calculated from the vertical separation of the centre of volume of the water body and its center of mass. Density stratification results in a lowering of the centre of mass by the vertical distance Δh_M (Figure 1) and the energy needed to overcome the stratification and to mix the entire water column is $\Delta E_{pot} = H\rho g \Delta h_M$ ($\mathrm{J\,m^{-2}}$), where H is the average depth of the water body, g is the gravitational acceleration, and ρ is the density. Density stratification thus imposes stability on the water column and reduces – or even suppresses – vertical mixing.

Besides convective mixing in the SL – caused by seasonal or nocturnal surface cooling – in most lakes and reservoirs, the major source of energy for vertical mixing is the wind, whereas river inflows usually play a minor role (Figure 1). As water is 800 times denser than air and as momentum is conserved across the air–water interface, SLs receive only about 3.5% of the wind energy from the atmosphere above. Surface waves transport a portion of this energy to the shore where it is dissipated; the remaining energy causes large-scale currents, with surface water flows of 1.5–3% of the wind velocity. Moreover, surface currents cause a stratified water body to pivot with warm water piling up at the downwind end (causing downwelling) and deep-water accumulating at the upwind end (causing upwelling). After the wind ceases, the water displacement relaxes and various internal waves develop – including basin-scale seiches – inducing motion even in the deepest layers.

These deep-water currents are usually one order of magnitude less energetic than those in the SL. Typical deep flows of a few centimeters per second (or $\sim 1\,\mathrm{J\,m^{-3}}$) with energy dissipation of less than $1\,\mathrm{mW\,m^{-2}}$ are able to reduce the potential energy of the stratification by only $\sim$0.01–0.05 $\mathrm{mW\,m^{-2}}$. Compared to the potential energy stored in the stratification (order of $1000\,\mathrm{J\,m^{-2}}$; Figure 1) it would take much longer than one season to entirely mix a moderately deep lake. This implies that wind energy input (Figure 1) forms the vertical hypolimnion structure at times of weak stratification (beginning of the season), whereas the wind is not able to substantially change the vertical structure once the strong stratification is established. Therefore, in most regions on Earth, only very shallow waters (less than a few meters deep (such as Lake Balaton, Hungary) are found to be entirely nonstratified, even during the summer season. The majority of lakes and reservoirs deeper than a few meters are thus only 'partially' mixed to a limited depth, which is basically defining the SL. For those lakes that show a pronounced SL, its maintenance is mostly supported by night-time cooling. In this article, we focus on the 'limited' mixing below the SL, which occurs in the metalimnion and hypolimnion (Figure 1).

Density Stratification and Mixing – the Small Scale

The same concepts of stability and mixing – described in the preceding section for the entire water body – also apply locally within the water column for small-scale vertical mixing of stratified layers. Local stability of the density stratification is quantified by the Brunt-Väisälä frequency (also buoyancy frequency) N ($\mathrm{s^{-1}}$), defined by:

$$N^2 = -\frac{g}{\rho}\frac{\partial \rho}{\partial z} \qquad [1]$$

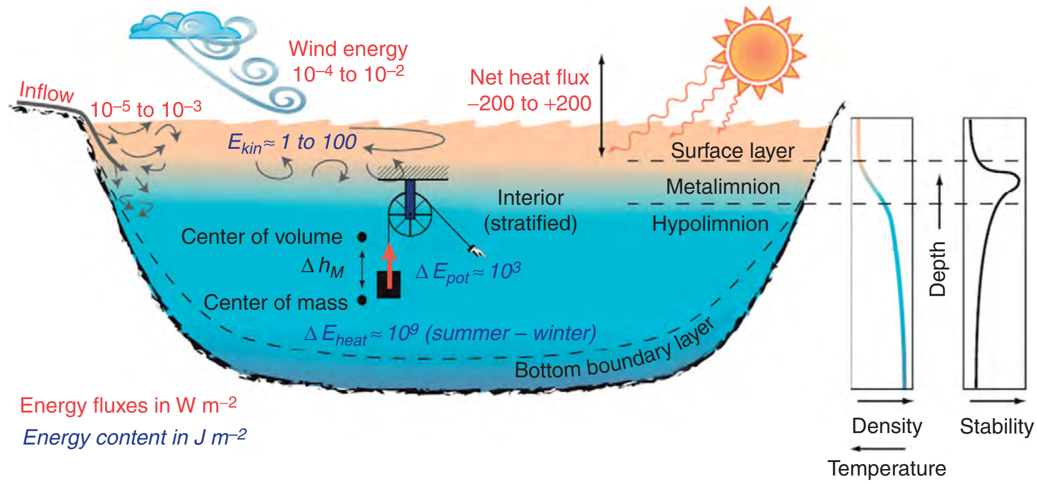

Figure 1 Energy fluxes (heat, wind, and river inflow; in red) into the water (W m^{-2}) and energy content (heat, kinetic energy, potential energy; in blue) stored in the lake water body (J m^{-2}). Note that the energy fluxes and contents related to heat are many orders of magnitude larger than those of kinetic and potential energy. The effect of mixing by the river is only local and less effective than wind. The stratified part of the lake (below surface layer) has historically been divided into the metalimnion (see large stability, right) and the deep hypolimnion (weak stratification) below. The lower water column can also be differentiated into an interior region (away from the boundaries) which is quiescent except during storms and a bottom boundary layer where turbulence is enhanced. Adopted from Imboden DM and Wüest A (1995) Mixing mechanisms in lakes. In: Lerman A, Imboden D, and Gat JR (eds.) *Physics and Chemistry of Lakes*, vol. 2, pp. 83–138. Berlin: Springer-Verlag.

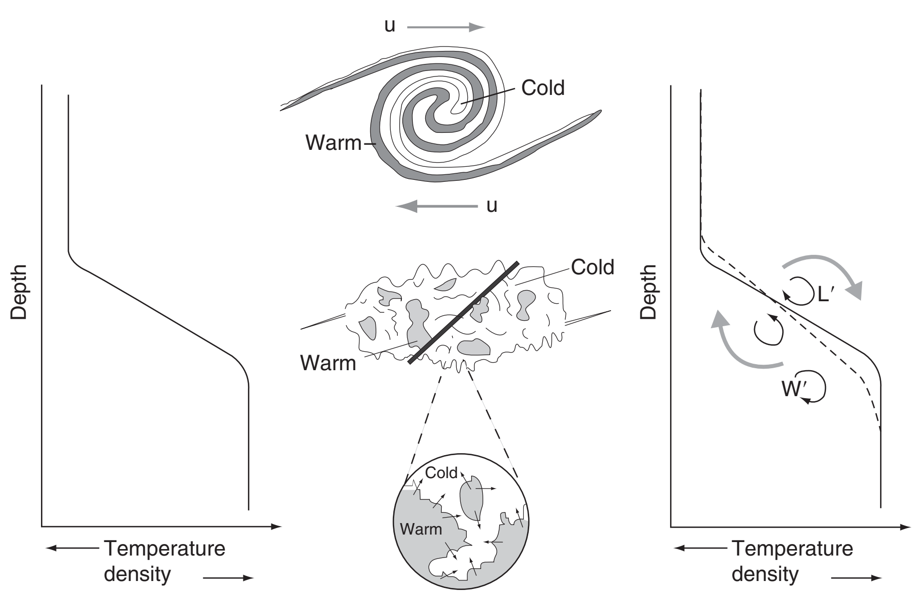

Figure 2 The effect of turbulent mixing in a stable stratification: if the vertical gradient of horizontal currents (current shear $\partial u/\partial z$) is stronger then the stability of the water column (eqn. [1]), Kelvin Helmholtz instabilities can develop (top of middle panel) bringing warmer (lighter) and cooler (heavier) water in close proximity (bottom of middle panel). Finally, heat (or any other water constituent) is mixed by molecular diffusion across the manifold small-scale interfaces, which are generated by turbulence. The turbulent exchange of small water parcels leads to a fluctuating vertical heat flux (see example in **Figure 3**) which averages to a net downward heat flux. As a result, the original temperature profile (left) is modified (right): the gradient is weakened and expanded vertically with heat transported from top to bottom, and density vice versa, across the interface. Figure after the idea of Winters KB, Lombard PN, Riley JJ, and D'asaro EA (1995) Available potential-energy and mixing in density-stratified fluids. *Journal of Fluid Mechanics* 289: 115–128. Experiments were first performed by Thorpe SA (1973) Experiments on instability and turbulence in a stratified shear-flow. *Journal of Fluid Mechanics* 61: 731–751; and the phenomenon of sheared stratification in lakes was reported by Mortimer CH (1952) Water movements in lakes during summer stratification; evidence from the distribution of temperature in Windermere. *Philosophical Transactions of the Royal Society of London B: Biological Sciences* 236(635): 355–398; and by Thorpe SA (1977) Turbulence and Mixing in a Scottish loch. *Philosophical Transactions of the Royal Society of London A: Mathematical Physics and Engineering Sciences* 286(1334): 125–181.

z is the depth (positive upward). As a result of wind-forced motions, a vertical gradient of the horizontal current u (shear $\partial u/\partial z$) is superimposed on the vertical density gradient $\partial\rho/\partial z$. Depending on the relative strength of N compared to the current shear $\partial u/\partial z$ such a stratified shear flow may eventually become unstable and develop into turbulence (Figure 2).

Although the large-scale (advective) motions are mainly horizontal, the turbulent eddies are associated with random velocity fluctuations in all three dimensions (u', v', w'). Turbulent kinetic energy (TKE) ($\mathrm{J\,kg^{-1}}$) is defined as the energy per unit mass of water which is contained in these velocity fluctuations:

$$\mathrm{TKE} = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) \qquad [2]$$

In stratified turbulence, the vertical velocity fluctuations w' are of particular importance as they transport water parcels and their contents in the vertical direction (Figure 3). The product of the vertical velocity fluctuations w' and the associated density fluctuations ρ' describes an instantaneous vertical flux of density ($w'\rho'$ ($\mathrm{kg\,m^{-2}\,s^{-1}}$)). Resulting from many irregular and uncorrelated fluctuations (Figure 3) the averaged flux $\overline{w'\rho'}$ leads to a net upward mass flux, which is usually expressed as a buoyancy flux J_b:

$$J_b = -\frac{g}{\rho}\overline{w'\rho'} \qquad [3]$$

Therefore, we can interpret vertical mixing as an upward flux of mass, which causes a change of the potential energy of the stratification (Figures 1 and 2), expressed as a buoyancy flux (eqn. [3]). The required energy originates from the TKE, which is itself extracted from the mean (horizontal) flow. Approximately 90% of the TKE, however, does not contribute to the buoyancy flux (and hence to vertical mixing) but is instead dissipated into heat by viscous friction, without any further effect. By defining local rates of production P ($\mathrm{W\,kg^{-1}}$) and viscous dissipation ε($\mathrm{W\,kg^{-1}}$) of TKE, the simplest form of TKE balance can be formulated as:

$$\frac{\partial}{\partial t}\mathrm{TKE} = P - \varepsilon - \mathcal{J}_b \qquad [4]$$

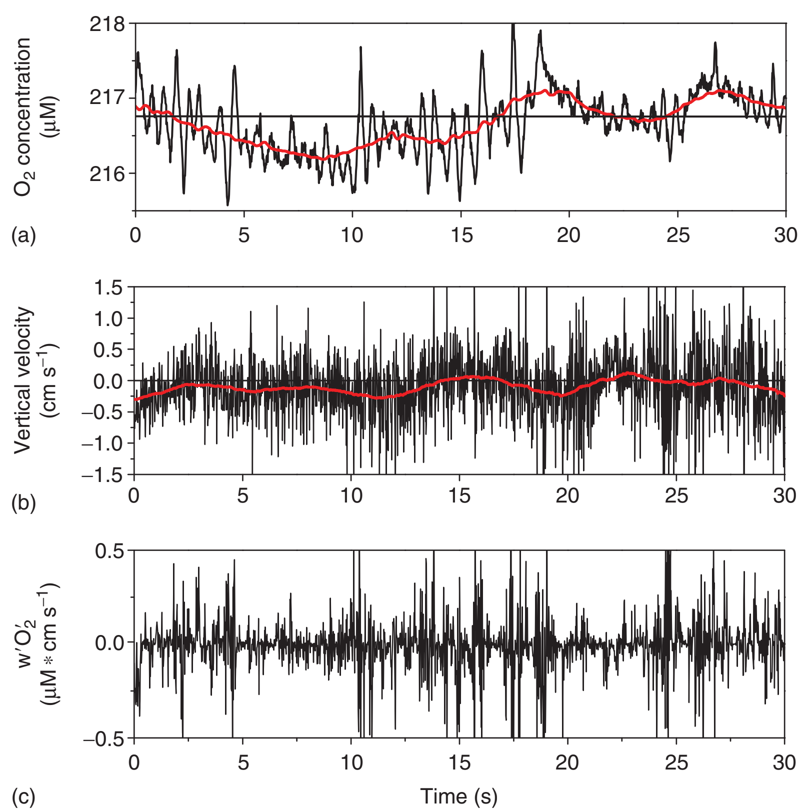

Figure 3 Time series of O_2 concentration (thin line, a) and vertical velocity w' (thin line, b; positive = upward), as measured 10 cm above the sediment in reservoir Wohlensee (Switzerland) at a frequency of 64 Hz. Red lines indicate the temporally varying averages, determined as running mean, whereas the black horizontal line marks the averages. Panel (c) shows the instantaneous eddy flux – covariance of w' and O_2': The average downward O_2 flux over the 30 s (~1900 data pairs) is −6.4 $\mathrm{mmol\,m^{-2}\,day^{-1}}$. Data source: Claudia Lorrai, Eawag.

As mentioned above, the dissipation rate is usually much larger than the buoyancy flux, and hence the mixing efficiency γ_{mix}, which is defined as the ratio

$$\gamma_{\text{mix}} = \frac{\mathcal{J}_b}{\varepsilon} = \frac{-g\rho^{-1}\overline{w'\rho'}}{\varepsilon} \quad [5]$$

is much smaller than 1. A number of studies in stratified lakes and reservoirs have revealed typical mixing efficiencies in the range of 10–15%.

Density Stratification and Mixing – the Turbulent Transport

The local flux of a water constituent is given by the product of the velocity times the concentration. In stratified waters, the time-averaged vertical velocity is often close to zero (negligible) and thus, the vertical fluxes stem only from the fluctuations of velocity and concentration, such as explained above for the vertical mass flux $\overline{w'\rho'}$ caused by the turbulence. This concept holds for any other water constituent, such as for oxygen, as exemplified in Figure 3, where the in situ measured w', O_2' and the product $w'O_2'$ is shown for a 30-s-long record. Although the momentary fluxes up and down are almost of equal variations and amounts, the averaging $\overline{w'O_2'}$ reveals slightly larger fluxes downwards to the sediment, where the oxygen is consumed.

Until recently, direct measurements of turbulent fluxes had not been possible and therefore turbulent fluxes in stratified waters are commonly expressed using the eddy diffusivity concept. Applied to the mass flux $\overline{w'\rho'}$ it implies assuming that (i) a well-defined local density gradient $\partial\rho/\partial z$ exists (due to the stratification) and (ii) the flux – in analogy to molecular diffusion – can be expressed by the eddy (or turbulent) diffusivity K_z ($m^2\,s^{-1}$) multiplied by this local gradient:

$$\overline{w'\rho'} = -K_z\frac{\partial\rho}{\partial z} \quad [6]$$

In this formulation, K_z describes the vertical transport of density caused by turbulent velocity fluctuations w' over a typical eddy distance L' given by the level of turbulence and the strength of the stratification. Therefore, in contrast to the molecular diffusion process, eddy diffusivity is neither a function of medium (water) nor of the water constituents (particulate or dissolved), but rather a property of the turbulent flow within the stratified water itself. In particular, K_z reflects the extent of the velocity fluctuations w' and the eddy sizes L': K_z can be interpreted as the statistical average $\overline{w'L'}$ of a large number of eddies, which exchange small water parcels as a result of the turbulent flow (Figures 2 and 3).

In addition to density, all other water properties – such as temperature or substances – are transported and mixed in the same way via the turbulent exchange of small eddies or parcels of water (Figure 3). The eddy diffusivity concept can be applied to any dissolved or particulate substance and the associated vertical fluxes F can be readily estimated in analogy to eqn. [6] by

$$F = -K_z\partial C/\partial z \quad [7]$$

where C is the appropriate concentration.

Assuming steady-state conditions, i.e., by neglecting the left-hand side of eqn. [4], and combining eqns. [1], [5], and [6] yields:

$$K_z = \gamma_{\text{mix}}\frac{\varepsilon}{N^2} \quad [8]$$

This equation provides an expression to estimate K_z from field measurements of ε and N^2 and, moreover, it demonstrates the direct proportionality of K_z on the level of turbulence (ε) and the inverse proportionality on the strength of stratification (N^2). In the last decades, two fundamentally different approaches have been used for the estimation of K_z: (i) the microstructure method and (ii) the tracer method. Method (i), is based on eqn. [8] where the dissipation of TKE or of temperature variations are measured by usually free-falling profilers which measure either temperature or velocity over small spatial scales. For example, spectral analysis of the temperature gradient signal provides estimates of ε and the local buoyancy frequency is obtained from density computed from the temperature and salinity profiles. For the application of tracer method (ii), one has to measure the three-dimensional spreading of a tracer (artificial or natural) and then infer the diffusivities (K_z) from the observations. Heat is also used as a tracer and K_z is obtained by computing a time series of the heat budget below the respective depth of a lake. Typical values in stratified natural waters are listed in Table 1 and Figures 4 and 5.

Turbulence is caused by current shear, breaking surface waves, and instabilities in the internal wave field. Currents induce shear near boundaries regardless of whether the flow is stratified. Thus, the concept of eddy diffusivity is also applied to surface mixing layers and to nonstratified systems such as rivers.

Turbulence and Mixing in Stratified Lakes and Reservoirs

Turbulence Production in the Surface and Bottom Boundaries

There are fundamentally two mechanisms generating turbulence in the SL: (i) the action of wind causing

Table 1 Typical values of dissipation, stability and vertical diffusivity in stratified waters

	Dissipation[a] *ε (W kg^{-1})*	*Stability N^2 (s^{-2})*	*Diffusivity* K_z (m^2 s^{-1})*
Ocean thermocline	10^{-10}–10^{-8}	$\sim10^{-4}$	$(0.3–3)\times10^{-5}$
Surface layer	10^{-6}–10^{-9}	0–$\sim10^{-5}$	10^{-5}–10^{-2}
Lake interior only (without BBL)	10^{-12}–10^{-10}	10^{-8}–10^{-3}	10^{-7}–10^{-5}
Metalimnion (basin scale)	10^{-10}–10^{-8}	$\sim10^{-3}$	$(0.5–50)\times10^{-7}$
Near-shore metalimnion	10^{-10}–10^{-6}	$\sim10^{-3}$	$(0.3–3)\times10^{-4}$
Deep hypolimnion (basin scale)	10^{-12}–10^{-10}	10^{-8}–10^{-6}	$(0.03–3)\times10^{-4}$

[a]During storm events values are larger by orders of magnitudes for short.

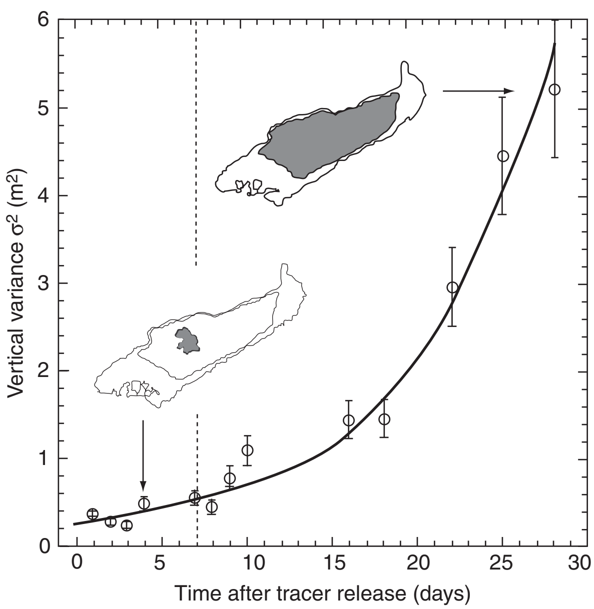

Figure 4 Vertical spreading of the tracer Uranine after injection at 25 m depth in Lake Alpnach (Switzerland). The vertical line demarcates the initial period of 7 days, during which Uranine resided in the interior of the stratified deep water. The two insets show the lake area at the surface and at the depth of the Uranine injection, as well as the horizontal distribution of the Uranine cloud (shaded in gray) after 4 and 28 days. The slow growth of the spreading in the first 7 days illustrates the quietness in the interior. The fast growth of the vertical spreading after day 7 is due to the increasing contribution of BBL mixing after the tracer has reached the sediment at 25 m depth. Reproduced from Goudsmit GH, Peeters F, Gloor M, and Wüest A (1997) Boundary versus internal diapycnal mixing in stratified natural waters. *Journal of Geophysical Research* 102: 27903–27914, with permission from American Geophysical Union.

wave breaking and shear in the top few meters of the water column and (ii) surface cooling causing the sinking of heavier water parcels. Temperature-driven mixing (case (ii)) leads to homogenization of the SL and therefore to nonstratified conditions – at least for a few hours or days before heat fluxes from/to the atmosphere restratify the SL. This process is discussed in detail elsewhere in this encyclopedia. Only in shallow ponds or basins with relatively high through-flow will turbulence have other case-specific sources.

For wind-driven mixing (case (i)), the crucial parameter governing the dynamics of turbulence in the SL is the surface shear stress τ (N m^{-2}), the force per unit area exerted on the water by the wind. This stress is equal to the downward eddy-transport of horizontal momentum from the atmosphere. Part of τ is consumed in the acceleration and maintenance of waves (τ_{Wave}), whereas the remaining momentum flux τ_{SL} generates currents and turbulence in the SL. By assuming a constant stress across the air–water interface, the two momentum fluxes on the water side equal the total wind stress ($\tau = \tau_{\mathrm{SL}} + \tau_{\mathrm{Wave}}$).

Immediately below the waves, the momentum flux, τ_{SL}, drives the vertical profiles of horizontal velocity $u(z)$ in the SL. If the wind remains relatively constant for hours, quasi-steady-state conditions may develop in the SL: $u(z)$ then depicts the Law-of-the-Wall $\partial u/\partial z = u_*(\kappa z)^{-1} = (\tau_{\mathrm{SL}}/\rho)^{1/2}(\kappa z)^{-1}$, where $u_* = (\tau_{\mathrm{SL}}/\rho)^{1/2}$ is the frictional velocity and κ ($=0.41$) is the von Karman constant. Because the buoyancy flux in the SL (defined in eqn. [3]) is not a large contribution in eqn. [4], we can assume a balance between the production of TKE and the rate of viscous dissipation (ε) of TKE. This local balance between production and dissipation of turbulence determines the turbulence intensity as a function of depth throughout the SL. Under those assumptions, the dissipation

$$\varepsilon = (\tau_{\mathrm{SL}}/\rho)\,\partial u/\partial z = u_*{}^3(\kappa z)^{-1} \qquad [9]$$

is only a function of the wind-induced stress τ_{SL} (here expressed as u_*) and of depth z. Several experiments have demonstrated that dissipation is indeed inversely proportional to depth (eqn. [9]), if averaged for long enough. However, one has to be critical about the validity of eqn. [9] for two reasons: First, at the very top of the water column, breaking waves, in addition to shear stress, produce a significant part of the turbulence in the SL. This additional TKE generation at the surface can be interpreted as an injection of TKE from above. Therefore, in the uppermost layer, the turbulence exceeds the level described by eqn. [9], depending on the intensity of the wave breaking. Second,

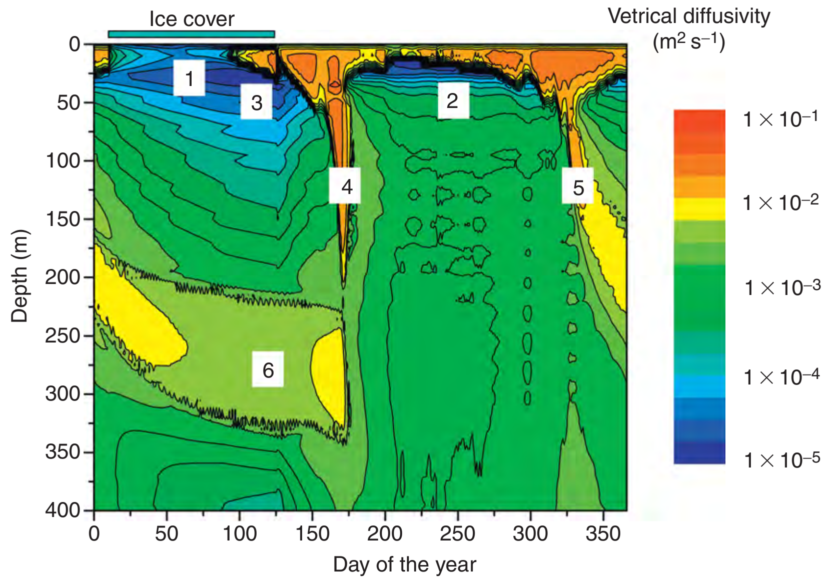

Figure 5 Vertical diffusivities in Lake Baikal simulated with a k-epsilon model. The numbers (1–6) on the contour plot indicate the main features of the seasonal stratification and changes in diffusivity: the formation of thermal stratification with weak mixing (1) during winter under the ice and (2) during summer; (3) the formation of a convectively mixed layer in spring under the ice; the deep convective mixing in (4) June and (5) November; and (6) the formation of a mixed layer near the temperature of maximum density. Here the emphasis is on the temporal and vertical structure of the turbulent diffusivity and not on the absolute accuracy, which may be difficult to achieve with turbulence modeling better than a factor of 2–3. Reproduced from Schmid M *et al*. (2007) Sources and sinks of methane in Lake Baikal: A synthesis of measurements and modeling. *Limnology and Oceanography 52:* 1824–1837, with permission from American Society of Liminology and Oceanography.

eqn. [9] relies on quasi-steady-state conditions which may hold applicable for limited episodes only.

Despite these restrictions, eqn. [9] gives a good estimate of the diffusivity in the SL, if it is weakly stratified. Equations [8] and [9] reveal that the rate of mixing increases substantially within the SL as the surface is approached. The corresponding stability N^2 decreases at the surface and maintains rapid mixing. Therefore, gradients of temperature, nutrients, and particulates are usually smallest at the surface and increase with depth. During sunny days, diurnal thermoclines form with mixing reduced below them. On cloudy, windy days, the SL may mix fully and may even deepen depending upon the surface forcing. Factors that affect the depth of mixing are discussed elsewhere in this encyclopedia. It is typically a few m during the warm season and a few tens of meters during the cold season. Below, a strong density gradient (pycnocline) can develop leading to the separation between the SL and the metalimnion/hypolimnion. In the stratified interior (away from the BBL; see below), the effect of wind is shielded and the mixing regime is completely different.

As discussed in greater detail (see **The Benthic Boundary Layer (in Rivers, Lakes and Reservoirs)**), turbulence generation and mixing along the bottom boundaries of water bodies can be described in analogy to the SL. Under steady-state conditions, the resulting bottom boundary layer (BBL) follows a similar vertical structure of (i) current shear (see above), (ii) rate of TKE dissipation (eqn. [9]) and (iii) rate of vertical mixing. Although the original indirect driving force for turbulence in the BBL is also the wind, it is not the direct turbulent momentum flux from the atmosphere to the water which is the cause. Rather, the mechanism is indirectly induced by wind which causes large-scale currents and basin-wide internal waves (such as seiches) which act as intermediate energy reservoirs that generate TKE by bottom friction. Along sloping boundaries in particular, the breaking of propagating internal waves and convective processes – a secondary effect of bottom friction – can produce additional TKE, leading to dissipation and mixing in excess of that predicted by eqn. [9]. As with the SL, the BBL is also usually partly (and weakly) stratified. Again, mixing (eqn. [8]) increases substantially when approaching the sediment and often a completely homogenized layer a few m thick develops at the bottom.

Internal Waves and Turbulence in the Stratified Interior

In the lake interior, away from surface and bottom boundaries (**Figure 1**), the water body is stratified and quiescent, and it does not feel the direct effects of the turbulence sources at the surface and above the sediment. This stratified interior consists of an upper region, the metalimnion where gradients in temperature and density are strongest, and a lower region, the hypolimnion, which is only weakly stratified and most water properties are homogeneous. Internal waves are prevalent.

The rate of mixing in the interior water body is low because (i) currents and shear are weak and the resulting turbulence production is reduced and (ii) stratification suppresses the turbulent mixing. The mechanical energy originates mainly from basin-scale internal currents and waves (see above), whereas the waves of smaller scale and higher frequencies – potentially generated at a few specific locations – are not contributing much to the energy budget of the deep-water. At the transition between small- and large-scale waves are the near-inertial currents, which can carry – especially in large lakes – a significant portion of the mechanical energy typically in the order of $\sim 1\,\mathrm{J\,m^{-3}}$. Given that observed energy residence time-scales are days (small lakes) to weeks (deepest lakes), the dissipation of the internal energy is $\sim 10^{-12}$–$\sim 10^{-10}\,\mathrm{W\,kg^{-1}}$ (**Table 1**). Considering typical values for stratification N^2 (eqn. [1]) of 10^{-8}–$10^{-3}\,\mathrm{s^{-2}}$ and $\gamma_{\mathrm{mix}} \approx 0.1$ (eqn. [5]), interior diffusivities of 10^{-7}–$10^{-5}\,\mathrm{m^2\,s^{-1}}$ can be expected (**Table 1**; **Figure 4**). The stratified interior – away from the SL and the BBL – is by far the most quite zone in lakes.

Important for the generation of small-scale mixing are local instabilities related to internal (baroclinic) motions, such as illustrated in **Figure 2**. Instabilities occur mostly where the usually weak background shear is enhanced by nonlinear steepening of internal waves or by superposition of the shear with small-scale propagating internal waves.

Direct observations of turbulence and mixing, using microstructure and tracer techniques, confirm that turbulence is indeed very weak in the stratified interior. Typically, only a few percent of the water column is found to be actively mixing. The occurrence of such turbulent patches is highly intermittent in space and time. During periods when the fluid is nonturbulent, we can expect laminar conditions and thus the dominance of molecular transport. The observable average diffusivity can be considered the superposition of a few turbulent events separated by molecular diffusion for most of the time. The resulting transport in the stratified interior will therefore be close to molecular. Tracer experiments and microstructure profiling conducted in small and medium-sized lakes confirm these quiet conditions in the interior and enhanced turbulence in the bottom boundary. In **Figure 4** the vertical spreading of a tracer, injected into the hypolimnion, is shown for the interior (first few days) and for a basin-wide volume including the BBL (after a few days). From **Figure 4** it is evident that turbulent diffusivity in the interior is at least one order of magnitude lower than in the basin-wide deep-water volume, including the bottom boundary. In addition to these spatial differences, one has to be aware of the temporal variability. During storms, turbulence can be several orders of magnitude larger for short episodes. The transition from quiescent to actively mixing occurs rapidly once winds increase above a certain threshold relative to the stratification. The internal wave field is energized and turbulence can develop. But the greatest increases occur in the benthic BBL. It is during such storms that most of the vertical flux takes place.

The turbulent patches, where vertical fluxes are generated (as exemplified in **Figure 3**) vary in size depending in part upon the turbulence intensity ε and the stratification N^2. Several length scales have been developed to characterize the sizes of turbulent eddies. One is the Ozmidov scale

$$L_{\mathrm{O}} = (\varepsilon/N^3)^{1/2} \qquad [10]$$

and the other is the Thorpe scale, L_{T}, which is based on direct observations of the size of unstable regions. The ratio of the two numbers varies depending upon the strength of stratification and is useful for predicting the efficiency of mixing, γ_{mix} in eqn. [5] Typical values for L_{O} and L_{T} range from a few centimeters to a meter but for weak stratification eddies are larger and on scales of tens of meters to 100 m as found in weakly stratified Lake Baikal.

The spatial and temporal dynamics of mixing challenges not only the experimental estimation, but also the numerical simulation of its net effect, in terms of a turbulent diffusivity K_z. Local measurements of K_z following eqn. [8] often neither resolve its spatial nor its temporal dynamics and the coarse grid sizes used in numerical simulations do not capture the small scales relevant for mixing processes in the interior.

Turbulent Energy Flux through the Water Column – Synthesis

From the discussion above, we can draw the following overall scheme of the energy flux through the stratified waters of a lake. The origin of the energy for turbulent mixing is usually wind, which is imposing momentum onto the surface of the water. Approximately 3% of the wind energy from the atmosphere reaches the epilimnion in the form of horizontal currents and about 10% thereof is finally transferred to the stratified water body underneath. The major part of the energy is dissipated by bottom interaction, and the minor part is dissipated in the interior by shear instabilities and breaking of internal waves. Of this dissipated energy, only about 10% produce buoyancy flux (mixing efficiency γ_{mix}, eqn. [5]) increasing the potential energy of the stratification. Compared to the wind energy flux in the atmosphere, only a small fraction of ~0.0003 actually causes the mixing against the stratification in the deep water, whereas the large fraction of ~0.9997 is dissipated somewhere along the flux path. Although this partitioning depends on many factors, the overall scheme likely holds within a factor of 2–3 based on comparisons between different lakes.

The small amount of energy available for mixing – compared to the potential energy stored in the stratification – explains why lakes deeper than a few meters remain permanently stratified during the warm season. Consistent with this conclusion, the enhanced turbulence in the surface and bottom boundary layers cannot erode the stable – and partly very strong – stratification in the interior. Turbulent patches are intermittent and the eddies within them are small compared to the depth. As an example, the timescales to transport heat, solutes or particulates over a distance of 10 m would be $(10\,\text{m})^2 K_z^{-1}$; i.e., several years for a $K_z = 10^{-6}\,\text{m}^2\,\text{s}^{-1}$ in the metalimnion (Table 1). Therefore, two-dimensional processes, such as upwelling become important for vertical exchanges as well.

See also: The Benthic Boundary Layer (in Rivers, Lakes and Reservoirs); Currents in Rivers; Currents in Stratified Water Bodies 1: Density-Driven Flows; Currents in Stratified Water Bodies 2: Internal Waves; Currents in Stratified Water Bodies 3: Effects of Rotation; Currents in the Upper Mixed Layer and in Unstratified Water Bodies; Density Stratification and Stability; Hydrodynamical Modeling; Mixing Dynamics in Lakes Across Climatic Zones; The Surface Mixed Layer in Lakes and Reservoirs; Water as a Human Resource.

Further Reading

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